Woke-Math never Forces Fixed Forms upon Flexible Totals

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Woke-math respects flexible bundle-numbers for a total instead of forcing a linear number upon it.

So, woke-math warns that imposing line-numbers without units as five upon a total of fingers will disrespect the fact that the actual total may exist in different forms, all with units: as 5 1s, as 1 5s, as 1Bundle3 2s, as 2Bundle1 2s, as 3Bundle-1 2s, etc.

Woke-math builds on a basic observation asking a 3year old "How old next time?" The answer typically is four showing four fingers. Holding them together 2 by 2, that child objects "That is not 4, that is 2 2s." The child thus sees what exist in space and time, bundles of 2s in space, and 2 of them in time when counted. So, what exist are totals to be counted and added in time and space, as T = 2 2s.

Woke-math thus builds on the philosophy called existentialism holding that existence precedes essence, i.e., that what exists outside precedes what we say about it inside.

By de-modeling mathematics instead of modeling reality, woke-math offers "Master Many to master Math" as an alternative to the traditional approach, "Master Math to master Many".

Woke-math respects, that outside totals inside may be counted and recounted in various two-dimensional bundle-numbers with units; and rejects one-dimensional line-numbers without units since they lead to 'mathematism' true inside, but seldom outside, where 2+3 = 5 is falsified by 2weeks + 3days = 17 days.

Counting and recounting totals in time and space produce flexible bundle-numbers

Counting in time produces a sequence of names like the names for the week days or months. Instead of saying "9, 10, 11 and 12", we might want to say "Bundle less 1, Bundle, one left, two left" to see the Viking origin of the names 'e-leven' and 'twe-lve', meaning precisely that.'

Later counting "10, 20, ..., 90, 100" may be paralleled be counting "1B, 2B, …, 9B, 10B or BB", thus fulfilling the Dienes dream of meeting power in grade one. That is also fulfilled when counting ten fingers in 3s as "0B1, 0B2, 0B3 or 1B0, …, 2B3 or 3B0 or 1BB0B0, 3B1 or 1BB0B1". With bundles and bundle-bundles, the place value system is not needed.

Counting in space, all totals have units. Some numbers are prime units, others are compound units hiding the prime units, 1 6s = 1x6 = 1x2x3 = 2 3s = 3 2s. Bundle-numbers become flexible by allowing overloads and underloads: T = 5 1s = 1B3 2s = 3B-1 2s.

Counting sticks in space we see that 4 1s may be rearranged as 1 4s, thus realizing that written less sloppy, digits are icons with as many sticks or strokes as they represent.

Counting 8 1s in 2s, pushing away 2s may be iconized by a broom called division allowing a calculator to predict the result: 8 = 8/2 2s. Stacking the 4 2s may be iconized by a lift called multiplication, 4x2.

So, we have a recount-formula, 8 = 8/2 2s = (8/2) x 2, or T = (T/B) x B with unspecified numbers, the core proportionality formula appearing all over math and science:

$\Delta y = (\Delta y/\Delta x) \times \Delta x = \text{slope} \times \Delta x$,

height = (height/base) x base = tan Angle x base,

meter = (meter/second) x second = speed x second, etc.

From the total, pulling away the stack to look for unbundled singles may be iconized as a rope called subtraction: 9 – 2x4 = 1. Placing the unbundled on-top of the stack introduces decimals, fractions, and negative numbers:

9 = 4.1 2s = 4 ½ 2s = 5.-1 2s.

Recounting a physical total in different units creates a per-number, as 4 $/5kg.

To change units, we just recount in the per-number: 20kg = (20/5) x 5kg = (20/5) x 4$= 16$

Recounting in the same unit creates fractions or percentages: 6$/20$ = 6/20 = 30/100 = 30%.

With prime-units, per-numbers may be simplified: 6/10 = (2x3) / (2x5) = (3 2s) / (5 2s) = 3/5

On a tile, recounting the sides in the diagonal creates trigonometry:

height = (height/base) x base = tan Angle x base, height = (height/diagonal) x diag. = sin Angle x diag.

This allows the number pi to be calculated as n x tan(180/n) for n large.

Adding totals on-top and next-to

Once counted and recounted, totals may add on-top after recounting makes the units like. Or next-to as areas called integration combining multiplication and addition, becoming differentiation combining subtraction and division when reversed, asking, e.g., 2 3s + ? 5s = 4 8s. Here, we remove the initial total to recount the rest in 5s:? = (4 8s – 2 3s) / 5 5s = Δy/Δx.

Per-numbers add by areas since adding 2kg at 3$/kg and 4kg at 5$/kg, the unit-numbers 2 and 4 add directly, whereas the per-numbers 3 and 5 must be multiplied to unit-numbers before adding, thus adding as areas.

Such mixture problems lead directly the 2x2 algebra square re-uniting changing and constant unit-numbers and per-numbers, where algebra in Arabic means 'to re-unite':

Changing unit-numbers unite by addition, and constant unit-numbers unite by multiplication. Likewise, changing per-numbers unite by integration, and constant per-numbers unite by power (10% + 10% = 21% since 110%^2 = 121%).

Uniting reversed will split a total in changing or constant unit-numbers or per-numbers. Here subtraction and division split a total into changing and constant unit-numbers. Differentiation splits a total into changing per-numbers. And finally, a total is split into constant factors by the factor-finding root and the factor-counting logarithm.

Bundle-numbers ease operations

Addition: T = 54 + 38 = 5B4 + 3B8 = 8B12 = 9B2 = 92 = 9.2 tens

Subtraction: T = 54 – 38 = 5B4 + 3B8 = 2B-4 = 1B6 = 16 = 1.6 tens

Division: 378 / 7 = 37B8 / 7 = 35B28 / 7 = 5B4 = 54 = 5.4 tens.

Multiplication: 6 x 7 = 6 x (B-3) = 6B -18 = (6-2)B (20-18) = 4B2 = 42 = 4.2 tens.

Multiplication tables as "6 7s = ?tens" may also be placed on a BxB square wring 6 and 7 with underload as B-4 and B-3.

So, 6 7s = 6x7 = (B-4) x (B-3) = BB -4B -3B + 4 3s (removed twice) = 3B 12 = 4B2 = 42 = 4.2 tens.

7 26s = 7 x 26 = 7 x 2B6 = 14B42 = 18B2 = 182 = 18.2 tens.

37 26s = 37 x 26 = (3B7) x (2B6) = 6BB (3x6 + 7x2)B 42 = 6BB 32B 42 = 6BB 36B 2 = 9BB 6B 2 = 962, seen on a 3B7 x 2B6 tile.

To find the square root of 40 we squeeze a 40 x 1 or 5 x 8 tile into a square. Here 6x6 is the largest inside square leaving 40 - 6x6 = 4 outside. Extending the 6x6 square into a (6+t) x (6+t) square adds additional two 6 x t tiles plus a t x t square that we neglect. With 6xt = 4/2, t = 2/6 = 0.333. So, the 40x1 tile squeezes into a 6.333 x 6.333 square approximately, or a little less since we neglect the t x t square. A calculator gives the answer: the square root of 40 is 6.325. Extending a square also shows that d(x^2) = 2x dx, or (x^2)' = 2x.

To solve the quadratic equation x^2 + 6x +8 = 0, we look at a (x+3) x (x+3) square divided in four sections, x^2, and 3x twice, and 9 = 8+1. Since x^2 + 6x +8 = 0, (x+3) squared is 1 = 1 squared. Hence x+3 = +1 or -1, so there are two solutions, x = -3+1 = -2, and x = -3-1 = -4.

References

MrAlTarp YouTube videos, especially "Flexible Bundle Numbers Develop the Childs Innate Mastery of Many", https://youtu.be/z_FM3Mm5RmE

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